(c) Show that $x = 2e^{5t}$, $y = e^{5t}$ and $x = e^{-t}$, $y = -e^{-t}$ are two linearly independent solutions on every interval $a \le t \le b$ of the homogeneous linear system :

$$\frac{dx}{dt} = 3x + 4y$$
$$\frac{dy}{dt} = 2x + y$$

Also, write the general solution.

5. (a) Form partial differential equation by eliminating arbitrary constants a and b from the equation

$$2z = (ax + y)^2 + b$$

(b) Find the general solution of the partial differential equation

$$(x - y)y^2u_x + (x - y)x^2u_y = (x^2 + y^2)u$$

(c) Solve the Cauchy problem

$$u_x + xu_y = 0, \quad u(0, y) = \sin y$$

6. (a) Solve

$$y^2u\frac{\partial u}{\partial x} + u^2x\frac{\partial u}{\partial y} = -xy^2$$

(b) Use $v = \ln u$ and then $v(x, y) = f(x) + g(y)$ to solve the equation

$$x^2u_x^2 + y^2u_y^2 = u^2$$

using the method of separation of variables.

(c) Reduce the equation: $u_x + xu_y = y$ to canonical form and hence find the general solution.

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 7045 **K**

Unique Paper Code : 2352572301

Name of the Paper : Differential Equations

Name of the Course : **B.A. / B.Sc. (Prog.) with Mathematics as Non-Major/ Minor**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt all questions by selecting two parts from each question.
3. **All** questions carry equal marks.

1. (a) Determine the most general function N(x, y) such that the following equation is exact, and solve the resulting exact equation:

$$(x^3 + xy^2)dx + N(x, y)dy = 0$$

(b) Solve the differential equation

$y^2\,dx - (1 - 3xy)\,dy = 0$

(c) Solve die differential equation

$(2x + \tan y)dx + (x - x^2 \tan y)\,dy = 0$

2. (a) Find the orthogonal trajectories of the family of circles which are tangent to the y axis at the origin.

(b) A large tank initially contains 100 gal of brine in which 10 lb of salt is dissolved. Starting at $t = 0$, pure water flows into the tank at the rate of 5 gal/min. The mixture is kept uniform by stirring and the well-stirred mixture leaves the tank at the rate of 2 gal/min. How much salt is in the tank at the end of 15 minutes and what is the concentration at that time?

(c) Consider the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0 .$$

(i) Show that e^x and xe^x linearly independent solutions of this equation on the interval $-\infty < x < +\infty$.

(ii) Find the solution that satisfies the condition $y(0) = 1$, $y'(0) = 4$. Explain why this solution is unique.

3. (a) Find the solution of the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0, \quad y(0) = -3, \quad y'(0) = -1.$$

(b) Using the method of undetermined coefficients, find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = \cos(4x)$$

(c) Using the method of Variation of Parameters, find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + y = \cot(x) .$$

4. (a) Find the general solution of the given differential equation

$$x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 4x - 6$$

(b) Find the general solution of the linear system

$$\frac{dx}{dt} + \frac{dy}{dt} - 2y - 4y = e^t$$
$$\frac{dx}{dt} + \frac{dy}{dt} - y = e^{4t}$$